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ICTE Journal

2019, **8**(1): 48–60
DOI: 10.2478/ijicte-2019-0005

INTERACTIVE LEARNING ENVIRONMENT SUPPORTING VISUALIZATION IN THE TEACHING OF PROBABILITY

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ABSTRACT

The probability is exceptional in the teaching of mathematics because students often have difficulties to understand the basic terms and the problem solving strategies. Understanding lacks of the probability concept and various types of misconceptions arise from the misleading intuition and misinterpretations of experience with the stochastic phenomena. The probability concept seems too abstract to some students therefore it is advisable to use mathematical problems based on real-life ideas, such as drug efficacy testing, tests for diagnosing of diseases in medicine, sports competitions, and games. By eliminating misconceptions and improving understanding of the problem solving strategies, it is possible to use various types of visualization to solve problems from this field of mathematics. Tables and different types of graphic diagrams can help students to understand the basic rules and problem solving techniques. This paper describes the main objectives and the structure of an interactive worksheet, prepared in spreadsheet environment, in which students are guided to use the visualization to solve probability problems. The implementation of an automatic feedback enables to evaluate the students' answers. In the case of incorrect answer, solutions of the additional tasks using tree diagram or a tangram are recommended to the students. Students can decide which type of the visualization is more understandable for them to solve the probability of random events. Solving different task sequences using the selected types of visualization allows more learning paths for students. The final part of the paper contains an evaluation of the results and experiences of problem solving in the teaching of probability at grammar schools.

KEYWORDS

Misconceptions, visualization, teaching of the probability, interactive worksheet, problem solving.

1 INTRODUCTION

Theme probability has in mathematics teaching exceptional position for several reasons. Students meet this difficult theme at secondary schools for the first time. Several authors (Garfield & Ahlgren, 1988; Knejpová & Vrábelová, 2009; Peard, 1987; Shaughnessy, 1992) agree that the teaching of probability is a difficult task because it is full of paradoxes and the basic principles of probability may seem too abstract for students. It is also difficult for students to understand the concept of probability and its regularities because the results analysis is often contrary to students' convictions and intuitions. Misconceptions and sources of misunderstanding are sometimes passed on to students by teachers themselves, who place emphasis in teaching on the application of definitions and formulas.

In the textbooks, the theme Probability follows the theme Combinatorics. For this reason, in the teaching of probability, a combinatorial approach, emphasizing the application of combinatorial rules, does not lead to an understanding of the basic principles of probability theory. Shaughnessy (1977) recommends that the teaching of probability should be based on the practical group or individual students' activity focused on making random experiments and analysing the results obtained.

Effective ways and means for supporting the teaching of probability are offered by modern digital technologies (DT), which enable computer simulations to be performed on various probability problems (Simon, Aktinson & Shevokas, 1976, Weissglass & Cummings, 1991). Simulation and analysis of a large number of random experiments allows applying a statistical approach to the teaching of probability. The results of several studies and researches (Arcavi, 2003, Presmeg, 1986, Zahner & Corter, 2010, Zimmermann, Cunningham, 1991) show that modelling using visualization can help students to understand solving of different probability problems and thus eliminate misconceptions, which arise in their minds based on experience and intuition.

2 MISCONCEPTIONS IN PROBABILITY

The teaching of probability should not only focus on explaining concepts and mastering problem-solving strategies but should tend to create new ideas, experiences, and the development of the probability thinking (Shaughnessy, 1992). If students learn to analyse the causes of wrong thinking in solving tasks and explain the reasons of conflicts between their calculations and the correct results of tasks, they can successfully develop their probability thinking.

The reason for the misunderstanding of the probability concept and students' failure in problem solving are misconceptions which resulting from incorrectly created ideas, misunderstood concepts, relationships, and misapplied methods of problem solving. Misconceptions are a critical aspect of the process of acquiring knowledge and skills and development of problem solving skills in the teaching of probability (Shaughnessy, 1992). In some cases, combinatorial considerations and basic combinatorial rules may also be used to solve probability problems. Therefore, the classification of mistakes and misconceptions in probability can be based on sources of mistakes and misunderstandings in combinatorics. For example, Batanero, Navarro-Pelayo, and Godino (1997) developed a detailed system for mistake classification in combinatorics.

Several authors have dealt with an analysis of probability misconceptions. Fischbein and Schnarch (1997) described several probability misconceptions and developed the following classification of misconceptions:

- Representativeness – people estimate the likelihood of an event according to how well it represents its reality;
- Negative and positive recency effects – "the negative recency effect" or "the gambler's fallacy" is if someone tosses a coin four times and gets four heads may then believe that the fifth toss is more likely to be tails. However, the belief that the fifth toss is more likely to be heads is called "the positive recency effect";
- Simple and compound events – if two dice are rolled, obtaining two sixes has the same likelihood as obtaining a five and a six;
- Conjunction fallacy – the probability of a random event appears to be higher than the probability of the intersection of the same event with another;
- Effect of sample size – people tend to neglect the influence of the magnitude of a sample when estimating probabilities;
- Availability – probability is estimated by the ease with which instances can be brought to mind;
- Time-axis fallacy (also called the Falk phenomenon) – people are likely to answer incorrectly based on the principle that an event cannot act retroactively on its cause.

LaiHuat Ang and Masitah Shahrill (2014) also characterized other misconceptions such as:

- Equiprobability bias – people tend to assume that random events are equally probable by nature. They view the chances of getting different outcomes as equally likely events;
- Beliefs – people think, that eventual outcome of an event depends on a force, which is beyond their control. Sometimes this force is God or some other force such as wind, other times luck or wishes;
- Human control – people generalise the behaviour of random generators such as dice, coins and spinners. They think the results depend on how one throws or handles these different devices.

Tversky, Khaneman (1982) and Shaughnessy (1981) think that most misconceptions in probability are based on misconception of representativeness. Madsen (1995) has grouped misconceptions into three more general classes:

- Representativeness – when asked to identify an outcome that is 'most likely' to occur, people will choose an outcome that appears representative. For example, in a series of six coin flips, we know that all sequences of H's and T's are equally likely. Given a choice among several possible sequences, people will often choose an outcome such as HHTHTT as being more likely than HHHHHH.
- Availability, experience - in trying to determine if an event is likely or unlikely (probable or improbable), the response is based on how readily an example of the event comes to mind. For example, people may determine the probability of winning a lottery by trying to recall people they know who have won.
- Outcome approach – people view a probability as a means of predicting the outcome on a single trial and they do not view it in terms of relative frequency of occurrence. If a probability greater than 0,5 is assigned to an event, then people would say that the event "should" occur and if it does not occur, then the assigned probability is wrong. For example, if a weather forecast says that there is an 80% chance of rain and if it does not rain, people using this interpretation will say that the forecast was "wrong".

Students often do not think about logical deduction and reasoning of the problem solving process but they try to get the results of the tasks by applying one of the acquired rules by which they perform the calculations with the given numbers.

In the paper, we focused on evaluation the level of understanding of the basic rules for calculating the probability of random events. Understanding the addition and multiplication rule in probability is one of the basic capabilities that can be used to solve different real-life probability problems. We designed and developed an interactive worksheet to diagnose the level of understanding of these rules. The interactive worksheet provides visualization to students who have failed when calculating the probability of two random events to facilitate understanding of these rules and their application to solve tasks. The interactive worksheet is created using spreadsheet MS Excel. The automatic feedback implemented in the worksheet provides the evaluation of students' tasks solutions and, in case of an incorrect answer of the introductory task, it leads students to use the visualization in solving problems.

3 VISUALIZATION IN PROCESS OF SOLVING PROBABILITY TASKS

In some fields of mathematics (e.g. geometry), visualization is an indispensable part of the learning process and students' problem solutions. In other fields of mathematics, visualization may not be an important part of the process of acquiring knowledge, but can often be used as a means of solving problems (Presmeg, 2006). One of these areas of mathematics is, according to Zahner and Corter (2010), the probability where visualization can help a student not only to understand the task assignment but also to look for problem solving strategy. Ideas about relationships highlighted in the clear pictures and diagrams can be important to understand the concept of probability and solving problems. According to Güler and Çiltaş (Güler & Ciltaş, 2011), teaching practices focused on the use of visualization forms are important for developing students' ability to solve problems and for motivating students.

Multiple studies (De Hevia & Spelke, 2009; Edens & Potter, 2008; Uesaka, Manolo & Ichikawa, 2007; Zahner & Corter, 2010) point out that visualization helps to solve probability problems and there is appropriate to introduce visualization into the teaching of probability. Some authors argue that the students' misconceptions in probability can also be eliminated by the use of visualization forms, because these models allow visualizing the abstract probabilistic terms and relationships.

Zahner and Corter (2010) point out that, in order for visualization to be a useful tool in solving problems, due attention must be paid to selecting a suitable visualization form. These authors emphasize that inappropriately chosen visualization forms do not lead to higher success of students in solving problems. Novick and Hmelo (1994) consider the selection of a suitable visualization form in the context of a given task to be an important skill in solving probability problems and thus they confirm these conclusions. The use of visualization is also associated with the discovery of strategies used for solving mathematical problems. These problems should be reasonably difficult in order for visualization to be necessary and to have the desired effect.

Various forms of visualization can be used for explaining probabilistic concepts, rules, and solving probability problems: images, sketches, schemas for ordered listings of random experiment results; (contingency) tables; Venn diagrams; tree diagrams; circular diagrams; tangrams (unit squares); coordinate systems; stochastic node graphs. Various visualization forms have different structural aspects or properties that determine the range of their usability. For example, tree diagrams are naturally suited for solving sequential problems (e.g. multiple coin tosses, throw a dice) or for listing all possibilities, while tables or Venn diagrams are suitable for representing compound events. It turns out that selecting a suitable visualization form is a key element in solving a probability problem. In the paper, we focused on visualization forms such as tree diagram and tangram, which are useful for illustration of application of addition and multiplication probability rule in solving the probability problems.

4 CONCEPT AND DEVELOPMENT OF THE INTERACTIVE WORKSHEET

Interactivity and feedback are implemented through logical functions and macros in the interactive worksheet. Therefore, the proper functionality of active components requires permission to run the macros when opening a worksheet. Tasks are placed on separate sheets. The Evaluation button provides evaluation of student's answer. A student receives information about the correctness/inaccuracy of the answer and, if necessary, the instruction for his/her next activity. After closing the feedback window, a sheet with the following task or a sheet with the final evaluation of the student's work is displayed. The choice of the next task is determined by the correctness/incorrectness of the task solution on the active sheet. The implementation of feedback enables to create the individual learning paths. If a student does not master the multiplication rule for calculating the probability of random events, he/she must complete a supplementary explanatory part in which visualization is used to repeat and explain the multiplication rule in probability.

Creation of a worksheet concept required selection of two tasks (marked as IW1 and IW2) with increasing difficulty, in which the addition and multiplication rules for calculating the probability of random events should be used. An additional task is an introduction of the explanatory part, in which students are guided to use the visualization. When working on other tasks in the interactive worksheet, students can create pictures and diagrams on paper. This practice is a concrete application of a link-based approach of computer-aided mathematical learning and paper-based learning, which is emphasized by Hähkiöniemi (2013). The task 1 (IW1) allows the distribution of students according to their knowledge of the multiplication rule in probability and their ability to apply it in solution of a typical task. Solving this task requires calculating the probability of a random event that when spinning two roulettes, the sum 6 of the numbers comes out. The sum 6 can only occur if the first roulette will stop at the number 4 and the second roulette will stop at the number 2. Correct solving of the Task 1 ensures that the student can continue to a more difficult Task 2 (IW2). Figure 1 shows the response of the interactive worksheet to a typical student error by adding the probability of individual random events.

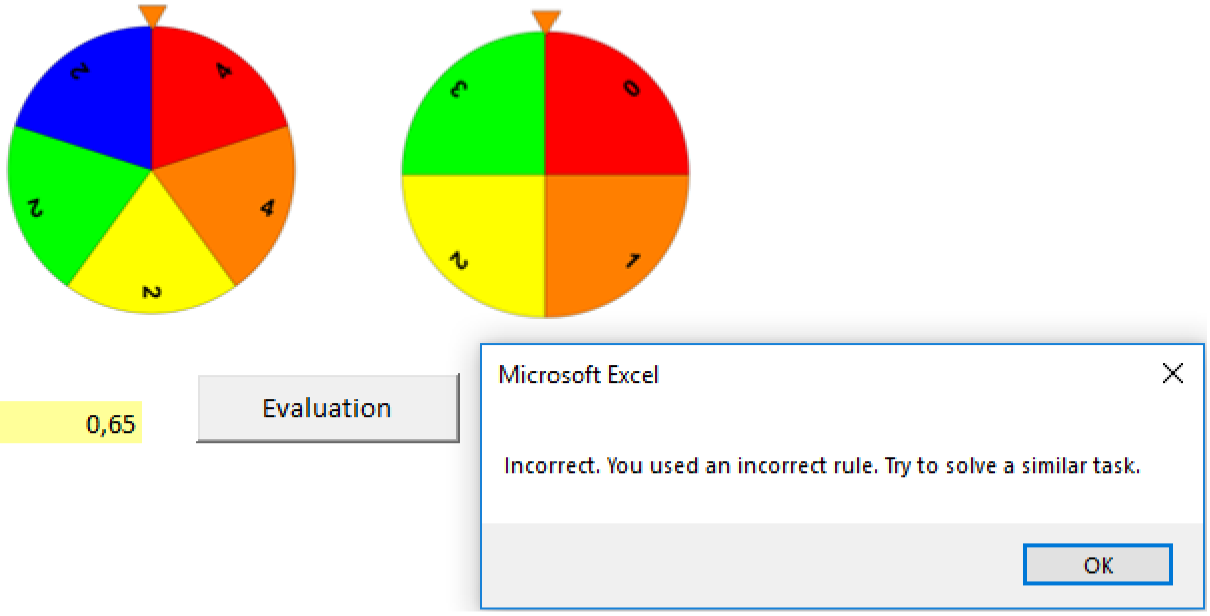

Figure 3 Evaluation of an incorrect answer in the Task 1

If a student solves the Task 1 incorrectly, he/she continues with the explanatory part, which contains the additional and control task. The additional task: *Specially trained dogs are at airports to check the presence of drugs in the luggage of the passengers. Dog A with a probability of 0,9 correctly determines whether drugs are hidden in the suitcase. Dog B with a probability of 0,85 correctly determines whether drugs are hidden in the suitcase. The customs officers will select the suitcase to check for dog A and then dog B. How likely is dog A and dog B to be mistaken when checking the suitcase?* The multiplication rule for calculating the probability of random events is explained in additional task using visualization. The part of the task is incomplete visualization forms such as tree diagram and tangram. The student can select a visualization form that is more comprehensible to him/ her, and append the result of an additional task (see Figure 2) to the yellow background cell in the selected diagram. If a student solves the additional task incorrectly, a final evaluation of his/her learning path will be displayed.

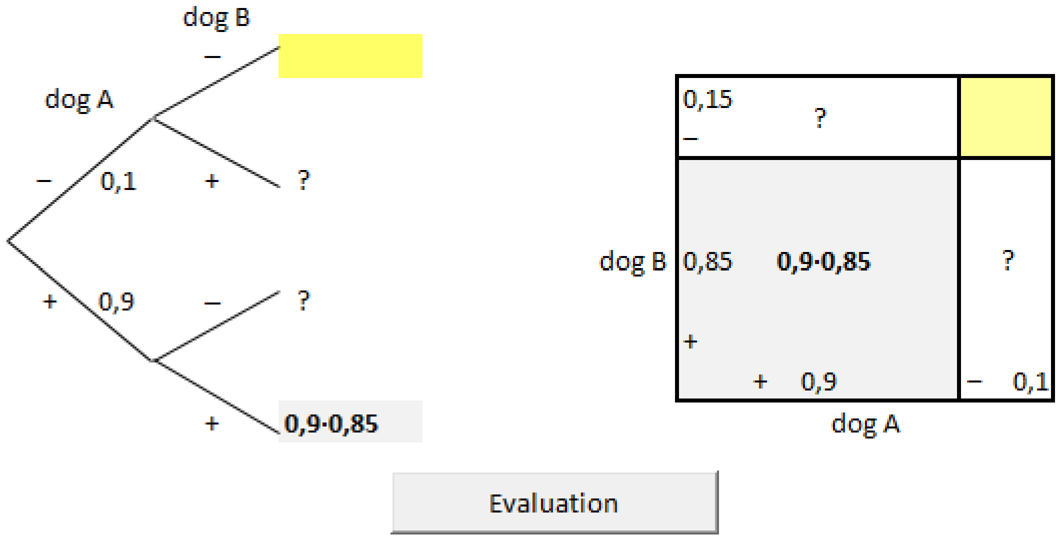

Figure 4 Visualization forms for solving the additional task

If a student solves the additional task correctly, he/she has to demonstrate understanding of the explained rule in solving of the control task. Only when the control task has been successfully solved, a student receives the assignment of the Task 2 (IW2): *The health organization estimates that 5% of the population has diabetes. Based on previous tests, they estimate that the medical laboratory's success in diagnosing people with diabetes is 0,98. In 2% of cases, the test in people with diabetes is negative. When testing healthy people, the success rate of the diagnostic test is 0,96. What is the likelihood that the result of a diagnostic test will be correct for a randomly selected person?* Solving this task requires the application of addition and multiplication rules for calculating the probability of random events. To solve this task, it is also advisable to use the visualization form. In this case, the student can already draw his/her own model on paper.

Finally, each student will get an evaluation sheet with the corresponding overall evaluation according to the learning outcomes. The evaluation allows dividing students into five groups. The first group consists of the best students who correctly solved the Task 1 as well as the Task 2 (or they did not solve the Task 1 correctly but then solved additional task, control task and the Task 2 correctly). In other groups, there are already students who have not received a positive evaluation. The evaluations of these students are indicated by letter N. We present them with an explanation of the relevant student learning outcomes.

N1: (The student did not solve the additional task correctly; he/she did not make the expected mistake of the incorrect use of the addition rule for calculating the probability of random events.) You did not understand the assignment of the task and its graphical interpretation. Ask for help from a teacher.

N2: (The student incorrectly used the addition rule for solving the additional task.) The random events occur independently of each other. Think about what rule is used to calculate the probability that both random events occur. Ask for help from a teacher.

N3: (The student solved the additional task correctly but did not solve the control task correctly.) You did not learn to solve the task calculating probability of two random events for which the probability of one event occurring does not affect the probability of a second random event. Ask for help from a teacher.

N4: (The student solved the Task 1 correctly but he/she solved the Task 2 incorrectly or the student solved the Task 1 incorrectly, but he/she solved the additional task and control task correctly and then he/she solved the Task 2 incorrectly.) You could not analyse the possible outcomes of random events and correctly apply the addition and multiplication rules in calculating the probability of two random events.

After completing the work with the interactive worksheet, the teacher will get a quick overview of students' knowledge and skills to solve the problems calculating the probability of random events based on the final evaluation. The teacher can use the obtained information for analysing students' solutions at the final stage of teaching. Students' results also provide the teacher a material for the formative assessment, including individual discussion with students leading to the detection of mistakes (Keeley & Tobey, 2011) and providing individual help to understand the basic rules for calculating the probability of random events.

5 PEDAGOGICAL EXPERIMENT TO DETECTION OF LEARNING RESULTS IN PROBABILITY

The main aim of our research is to find out whether students try to use visualization to solve probability tasks and how visualization helps them to find the solution of tasks. We focused on diagnosing the rate of acquisition of the basic rules used to determine the probability of random events at secondary school.

We have identified the following research questions for our research:

1. To what level did students know the addition rule and multiplication rule for calculating the probability of random events?
2. Which type of visualization form prefers the students to solve tasks, in which two random events occur?
3. How are students able to apply the acquired knowledge to solve probability problems?

We prepared two research tools in the form of the interactive worksheet and the printed worksheet to find answers to research questions. We do not force students to use different forms of visualization in the interactive worksheet. An explanatory part containing two forms of visualization: a tree diagram and a unit square. They were offered only to students who have failed in solving the first task. Students could decide independently whether they use visualization to solve tasks in the printed worksheet.

We chose one class from the third grade of grammar school for pilot research. The research sample consisted of 24 students. Students from the selected class finished the theme Probability approximately three weeks before the experimental lesson. Interviews with a teacher, who teaches mathematics in this class, pointed out that they were using tables and schemes for solving probability tasks using the listing of possibilities.

Tasks solutions in both worksheets were analysed from two aspects: the correctness of the solution and the way of the use of the visualization. According to small research sample, we focused on qualitative analysis of students' solutions. When analysing students' solutions, we focused on assessing the correctness of tasks solutions and on reviewing whether visualization helps students in solving the probability tasks.

The lesson lasted 90 minutes. Resolving tasks in the interactive worksheet required independent work of students at the computer. After completing the work with the interactive worksheet, the students started to solve tasks in the printed worksheet. The printed worksheet contained two main problems (marked as W2 and W4). Before these two problems, there were included auxiliary tasks (marked as W1 and W3) designed to guide students to solve the main problems. The auxiliary task for the first problem (W1) was very simple and we did not included it to overall evaluation. The problem (W2) was similar to the Task 2 (IW2) about diabetes diagnosis. It describes a game based on throwing two regular dodecahedrons, in which figures different numbers of different and same images of the animal. The students had to calculate the probability of player winning based on throwing the pictures of the same animals. Task solving could be based on listing of possibilities or creating a suitable visualization form.

To increase the interest of students, we tried to choose a problem with popularizing content. Problem 2 was preceded by an auxiliary task (W3): *Four cards with the number 1 and two cards with the number 2 are in the first bag. The second bag is empty. Peter took one card with the number 1 and one card with the number 2 and put it in an empty bag. John then randomly chose one bag and picked one card out of it. What is the probability that the card with the number 1?*

Problem 2 (W4): *Headsman brought two identical bags and ten white and ten black marbles to the prisoner. On the next morning, the headsman will choose one bag randomly and then he will pick one marble out of it. If it is white, they will grant a grace to the prisoner, if it is black, the prisoner will execute. How does a prisoner have to split the marbles into both bags so that neither bag remained empty and have the biggest chance of granting a grace? Calculate the likelihood that the prisoner will grant a grace for your distribution of marbles to the bags.*

The problem 2 is more difficult because the students had to find a distribution of the marbles into two bags, giving the prisoner the biggest chance of granting a grace and subsequently calculating the probability of picking a white marble from a randomly selected bag.

6 RESULTS AND DISCUSSION

We selected for evaluation the main tasks from both worksheets. The average students' success rates are presented in the following graph.

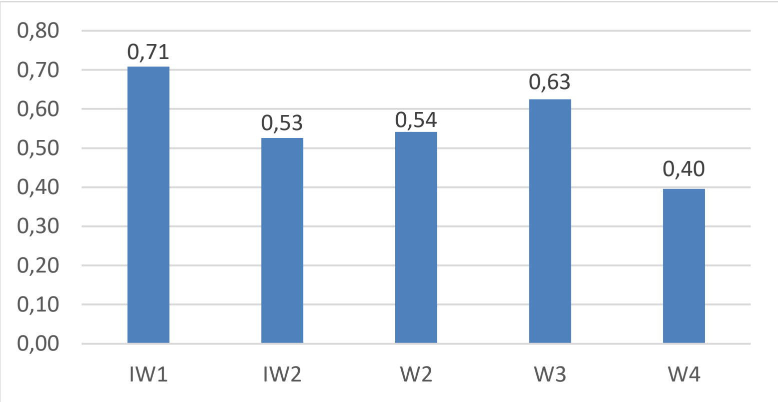

Figure 3 Average students' success rates

The results of the students' solutions in the interactive worksheet provided data to answer the first research question. Seven students solved Task 1 (IW1) incorrectly and therefore they had to go through an explanatory part of the interactive worksheet containing the additional and control task. Three students used the addition rule instead of the multiplication rule in solving this task. It shows that these students misunderstood the basic rules for calculating the probability of independent random events. Five of seven students solved the additional task correctly and two of these five students solved the control task correctly and that is why they could solve the task 2 (IW2). Only 19 students solved this task. From these results, it can be concluded that students have learned the basic rules for calculating the probability of random events at the average level.

When creating an explanatory part of the interactive worksheet, we assumed that students had already used some forms of visualization to solve combinatorial and probability problems. Therefore, the explanatory part was brief and it contains the two most frequently used forms of visualization in solving the tasks included in the worksheet. Students could choose visualization in the form of tree diagram or tangram. Five students chose a tree diagram and two students chose a tangram. The tree diagram allows making a clear representation of the possibilities that can occur, and the tangram clearly demonstrates the use of the multiplication rule in probability of random events. We think that the preference of the tree diagram is also affected by the fact that students already have had experience with using this diagram when they were solving combinatorial tasks. This tendency has also clearly emerged when they were solving tasks from the printed worksheet. When answering the second research question, we can say that students prefer the tree diagram. Although this diagram is suitable for analysing the results of random experiments, it is also appropriate to use a tangram, which connects multiplication rule of probability with calculating the rectangular area (Płocki, 2007).

All 24 students solved tasks in the printed worksheet. Problem 1 (W2) associated with throwing two dodecahedrons was presented to the students as a game. Students' success rate in solving this problem is comparable with the success rate of solving the task 2 (IW2).

Students also tried to use visualization in solving Problem 2 (W4) and the related auxiliary task (W3). Therefore, when evaluating the results of problem solving from the printed worksheet, we will focus on evaluating the students' solutions of Problem 2 and the related auxiliary task. Nine students used some form of visualization in solving the auxiliary task W3. Six students used a tree diagram and five of them solved the task correctly.

Solving the Problem 2 (W4) requires combining the basic addition and multiplication rule. Even seven students intuitively assumed that it would not be possible to find the distribution of the marbles in bags to make the probability of selecting a white marble greater than 0,5. This false supposition caused by the misconception called "Equiprobability bias" has greatly influenced students' solutions. In formulating the answer to the third research question, it can be concluded from the above results that students have difficulties to apply the basic probability rules in solving problems requiring analysis of several cases where the systematic listing of possibilities cannot be easily used. Lower student success in solving probability problems is often caused by misunderstanding of the problem and excessive reliance on intuition and experience from real life.

We evaluate the use of visualization and the students' success in solving tasks in the interactive and printed worksheet. Seven students solved the explanatory part but only two of them completed this part successfully. These two students solved the task 2 (IW2) only partially. These students did not understand the basic rules for calculating probability and the brief introduction to graphical expression of relations was not sufficient for them. When solving the tasks in the printed worksheet, students mostly used visualization to solve tasks W3 and W4. We divided students into two groups: students who did not use the visualization and students who use the visualization at least in one task. We calculated the average success rate for both groups in solving these tasks. Results are displayed in the following table (see Figure 4). It was shown that the less clever students achieved worse results although they tried to use the visualization forms more often. Clever students, who acquired basic rules for calculating the probability of random events, based their solutions on calculations using these rules without visualization.

Group	Number	Succes rate
Visualization	10	47,5%
No visualization	14	53,6%

Figure 4 Comparing students' success rate in created groups

In following qualitative analysis, we describe selected students' mistakes and ways of visualization in solving the tasks. Information from a mathematics teacher showed that various forms of visualization in combinatorics and probability teaching were used in the experimental class. Therefore, we expected that students would use the visualization to solve problems. In spite of our expectations, students used the visualization in a small extent, and they used from different visualization forms mainly tables and tree diagrams. Most students used tree diagrams to solve the auxiliary task to Problem 2. Especially students who have completed the explanatory part in the interactive worksheet have tried to use visualization to solve tasks in the printed worksheet. For example, we chose a tree diagram created by this student to solve the auxiliary task to Problem 2 (see Figure 5).

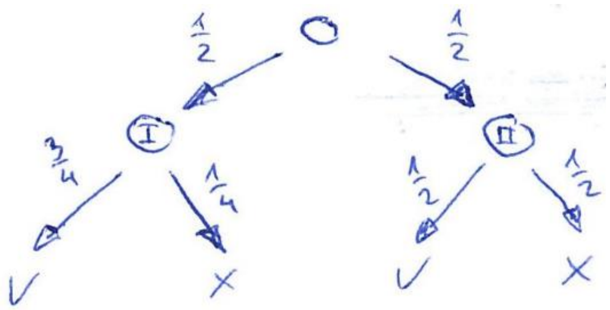

Figure 5 Student's tree diagram for solving the auxiliary task

Three students inappropriately used the addition rule instead of the multiplication rule in solving tasks in the interactive worksheet. The reason for this mistake is the misconception of a misunderstanding of the concept of basic rules (Batanero, Navarro-Pelayo & Godino, 1997). Being aware of this mistake after the brief explanations provided in the automatic feedback, such a mistake was no longer present in solving the tasks in the printed worksheet.

Visualization in solving Problem 2 was most related to distribution of marbles into bags. Several students plotted only one distribution of the marbles and formulated premature incorrect conclusions. Figure 6 shows a picture that was a part of the incorrect student solution.

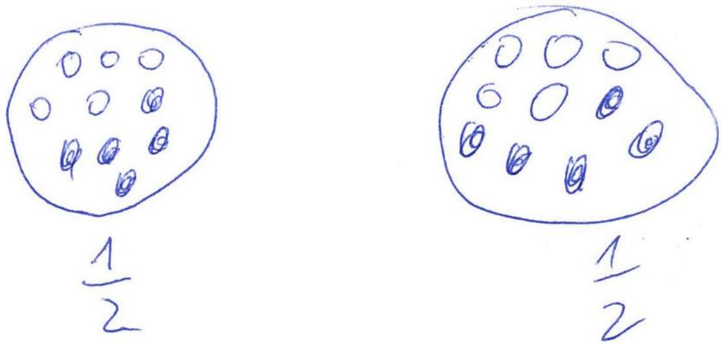

Figure 6 Student's illustration of one distribution of the marbles into the bags in the Problem 2

Three students created pictures, which only illustrated the situation described in the Problem 2. These students did not solve the problem correctly. As stated by Boonen, van Wesel, Jolles, and van der Schoot (2014), concentrating on the sketch of insignificant details can divert the students' attention in the wrong direction, and in many cases does not lead to a correct task solution. For example, we have selected one picture of this type (see Figure 7).

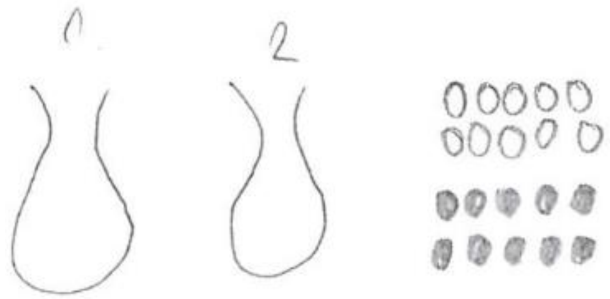

Figure 7 Student's illustration in solving the Problem 2

Analysis of students' solutions and ways of using visualization in problem solving can provide useful information for creating assessment tools that can be used for purposes of the formative assessment. Selected probability problems and misconceptions of students in their solutions can provide to mathematics teachers a basis for understanding student strategies of problem solving and providing feedback for effective student learning.

CONCLUSION

If students are not lead by a teacher to use the appropriate visualization forms, then their visualization often involves drawing the details of the subjects described in the problem. An illustration of unimportant information diverts the students' attention from the analysis and representation of the important relationships described in the problem. A tree diagram can be used appropriately in teaching of several topics in school mathematics. This type of a diagram allows illustrating clearly and systematically the various possibilities or cases that need to be analysed during problem solving. However, other forms of visualization may be used for solving different types of problems. The small research sample does not allow us to formulate general recommendations and conclusions. In view of our results and the conclusions of other studies, the use of various forms of visualization forms in the teaching of probability may be recommended to mathematics teachers.

We plan to continue in our research focused on the use of visualization in solving probability tasks. Based on described results, we replace the first task in the interactive worksheet with more difficult task to use the multiplication rule in calculating the probability of random events. We plan to extend the explanatory part with the example in which we will explain the use of the tree diagram and the unit square in solving the probability tasks. In our opinion, tangrams are appropriate for understanding and applying the multiplication rule in solving probability problems. Mathematics teachers at all levels of schools should consider which forms of visualization can help students to understand the problem solving strategy and how to integrate them into teaching of mathematics.

ACKNOWLEDGEMENTS

The paper was created with support of the VEGA project no. 1/0265/17: Formative assessment in science, mathematics and informatics.

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