2017, *6*(3): 26–39
DOI: 10.1515/ijicte-2017-0013

IMPLEMENTATION OF FEEDBACK SUPPORTING FORMATIVE ASSESSMENT INTO MATHEMATICAL DIGITAL LEARNING MATERIALS

Stanislav Lukáč[1], Jozef Sekerák[2]

[1]*Institute of Mathematics, Faculty of Science, Pavol Jozef Šafárik University in Košice, Slovak Republic {stanislav.lukac@upjs.sk}*

[2]*Lifelong Learning Centre and Projects Support, Pavol Jozef Šafárik University in Košice, Slovak Republic {jozef.sekerak@upjs.sk}*

ABSTRACT

Official documents of educational policy of the European Union highlight the importance of integrating ICT into education. Interactive learning materials play an important role in the meaningful use of ICT to support an active learning. Providing feedback dependent on student´s actions should be a standard part of the digital learning materials. The immediate feedback in the classroom during the learning process is in most cases provided by the teacher. The importance of feedback even increases when students work with the digital learning materials independently in the classroom or at home. Moreover, ICT tools offer a great potential for detecting and eliminating students' mistakes. A notification of a concrete type of error in student's way of task solution and guiding students during the corrections of their solutions creates conditions for application of formative assessment which provides suggestions to students for improving their learning. The effectiveness of feedback depends on the way how the typical errors of students are taken into account and what way the appropriate hints are implemented in learning. Various forms of feedback should be implemented in the interactive learning materials in order to stimulate the active learning. In many available the digital learning materials there is often a minimal feedback provided on student´s solution of a certain task. This paper discusses the implementation of various types of feedback in an interactive mathematics learning environment. Even though the minimal feedback can stimulate the active learning, it may not be sufficient for some types of students who do not think deeply about their methods of problem solving. Higher levels of feedback are characterized by providing comments that depend on student's mistakes and by providing pieces of advice and other helpful additional information that are closely related to the solution of tasks. The diagnosis of typical student´s misconceptions was based on the analysis of the results of the prepared tests, which were given to students at high school. Demonstrations of the minimal feedback and higher levels of feedback are illustrated in the examples of fractions, percentages and investigation of function dependencies. The interactive learning materials are developed using Geogebra and MS Excel.

KEYWORDS

Mathematics teaching, misconceptions, interactive learning materials, feedback, formative assessment, problem solving.

1 INTRODUCTION

The policy for mathematics education in Europe described in the publication (EACEA P9 Eurydice, 2011) emphasizes these attributes of the effective methods of mathematics teaching: conceptual understanding and interpretations of representations; learning strategies for investigation and problem solving. The innovative teaching methods in mathematics can be suitably supported by information and communication technologies (ICT). ICT enable students work with diagrams and graphs and develop connections between representations (Žilková, 2009). The computer simulations are designed to facilitate teaching and learning through visualization and interaction with dynamic models (Sarabando, Cravlno & Soares, 2016). Innovative approaches to mathematics teaching are often based on independent inquiry of mathematical relationships (Koreňová, 2015). The inquiry-based learning fosters observations followed by experimentation, modelling, and justification of findings (Hähkiöniemi, 2013).

The effect of inquiry-based learning on acquisition of knowledge and skills and on the understanding of the learning content depends on many factors. Jeff C. Marshall identified in his publication (Marshall, 2013) the three most important factors:

1. Learning lessons stimulating an active students' inquiry. Designed research questions should engage students and should be appropriate to their knowledge and abilities. Marshall recommends to use the model 5E (Engage, Explore, Explain, Elaborate, and Evaluate) for the organization of the inquiry based learning.
2. Formative assessment. Some teachers consider the assessment as the final stage of teaching. However, the assessment may also include various processes implemented during the teaching and provide students stimuli to improve their learning.
3. Teacher's reflexion. Collecting and analysing information about the learning success and the deficiencies that have occurred during the lesson provide the teacher bases for the retrospective evaluation of the learning process and the teaching materials, eventually also for their corrections.

Better understanding of findings discovered during inquiry and their elaboration to student's internal knowledge system can be enhanced by the formative assessment (Keeley & Tobey, 2011). The basis of the formative assessment is the analysis and use of information from a variety of interactions between teachers and students to provide feedback to students in order to improve their further learning.

The success of the independent student investigation depends strongly on the selection of the research question and student motivation (Rocard, 2007). Kyriacou and Goulding (EACEA P9 Eurydice, 2011) declare that ICT can raise student motivation for the active investigation and learning. They describe some surveys in Europe focused on ways of the use of ICT in mathematics education. Some results show that the use of ICT at home for school relate work is still relatively low. The digital learning materials, in which are implemented various levels of feedback, can suitably support home learning activities.

2 DESIGN OF FEEDBACK IN THE INTERACTIVE LEARNING MATERIALS

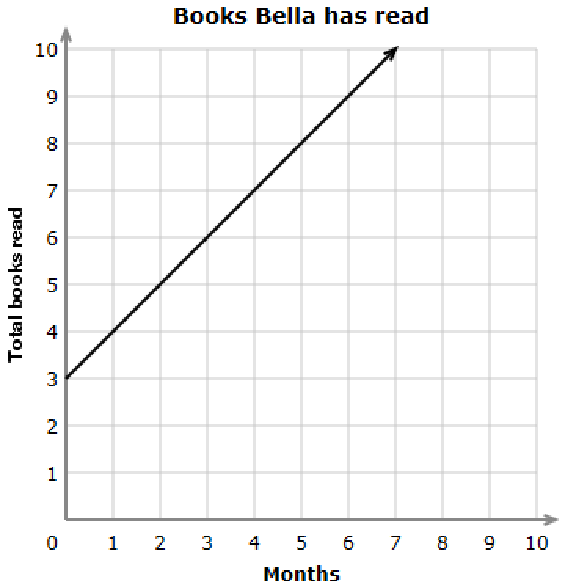
Feedback in the learning materials offers means for stimulation of an active learning. ICT can offer quick and contextual feedback providing students information on the correction of their results acquired through learning. Mathematics teachers have some resources of the digital learning materials available on the internet, such as The planet of knowledge, Maxi test, etc. Teachers can edit or create own learning materials in the both mentioned systems. Authors of various educational systems often pay little attention for providing of feedback. But implementation of monitoring and managing the learning and evaluation of results into the interactive learning materials is essential. In case of an incorrect results, feedback can encourage students to rethink their considerations and try to correct procedures for solving tasks. ICT provides possibilities to implement various types of feedback into the learning materials. Feedback in the learning materials can be implemented through several ways. McKendree (McKendree, 1990) sees main aim of feedback in the interactive learning materials in the following factors: student is called to try again to solve the task, student obtains hints and advices for actions, and student receives short explanation of his/her error. This author distinguished three basic levels of feedback:

1. Minimal feedback: student gets only brief information about the correctness or incorrectness of his/her answer.

2. Condition violation feedback: in the case of incorrect answer student is advised on incorrectly applied a rule.
3. Goal feedback: the information for student is formulated in such a way that it should help him/her to find the correct solution. Student is informed about requirements which are necessary for elimination of the mistake in next work.

Conclusions of research performed by Perrenet and Groen (Perrenet & Groen, 1993) show that hints explaining a great part of the problem solution seem to be effective, however the solution was often reached with lack of understanding. Feedback is more effective, if it stimulates concrete actions for the required solution method. Better educational content acquisition and increasing of chance to thoroughly learn and master curriculum require to clarify problematic issues of a particular curriculum (Prextová, 2015). Identifying a typical student's mistake or misconception would lead to providing a counterexample required next action or an auxiliary question suggesting a solution of the task. The well-known mathematician Polya dealt with problem solving. He emphasized the suitability of using a heuristic strategy based on reformulating the problem (Polya, 1957). If a student cannot solve the original problem, the system may offer a related or reformulated problem which presents a simpler problem for the student or a problem with which he/she should already have experience.

This graph shows how the total number of books Bella has read depends on the number of months she has been part of a book club.



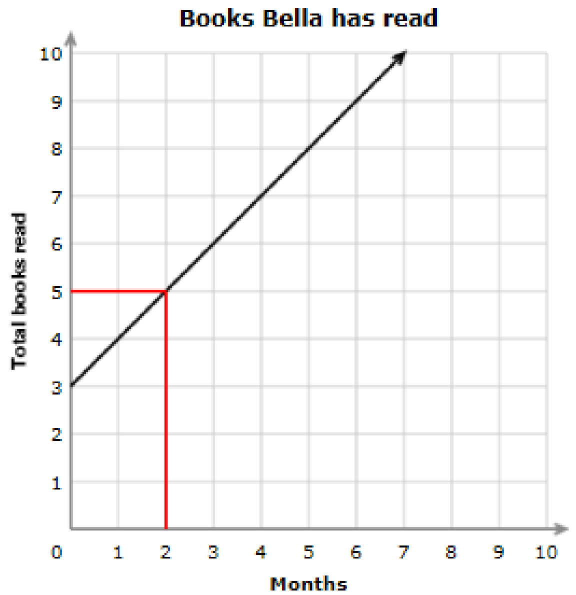
After belonging to the book club for 2 months, how many books will Bella have read in all?

books Submit

Figure 1 The assignment of the task

We will illustrate various types of providing of feedback in some digital learning systems. The learning system The planet of knowledge offers a modern learning environment for active acquisition of mathematical knowledge and skills. Interactive exercises in lessons often give students only minimal feedback about correctness of results. The response of the system consists of a graphic and audio alert. The lesson conclusion also includes a summary of the correctness of the exercise solutions. The interactive exercises available on www.math-tests.com provide not only minimal feedback about correctness of results but correct answer too. More extensive feedback is provided by the portal eu.ixl.com. For an example, we selected the word problem to interpret the graph of a linear function (see figure 1). In the event of an incorrect response, the system also provides the explanation of task solution (see figure 2).

Find 2 months on the *x*-axis. Move up until you intersect the graph.
Now move left until you intersect the *y*-axis.



You intersect the *y*-axis at 5 books. After belonging to the book club for 2 months, Bella will have read 5 books in all.

Figure 2 The explanation of the task solving

3 IDENTIFICATION AND ANALYSIS OF STUDENTS' ERRORS

During the planning learning process, the teacher considers the typical errors of students and looks for ways to detect these errors and to help students to master educational content. The teacher observes the work of students, questions them and analyses their answers. The teacher analyses the identified shortcomings and misconceptions and provides students appropriate examples as well as counterexamples supporting argumentation and developing critical thinking. Providing quick and effective feedback is a key factor in formative assessment. Students receive important information and guidelines from the teacher, where they made mistakes, how to correct their mistakes and how to improve their learning.

A major obstacle to acquisition and understanding curriculum is creation of deformed and faulty concepts, misunderstanding of basic contexts and methods of problem solving. These phenomena are the source of misconceptions and non-successes of students in problem solving. Another consequence is reflected in the reduction of interest in mathematics education. Therefore, it is important for the teacher to identify critical places in the learning content and to know the typical students' mistakes and misconceptions. In the paper, we focused on the selected basic parts of the elementary school mathematics: word problem focused on fractions and percentages and linear functions.

When students work with fractions, errors occur already in graphical representation of fractions. For fraction representations using circles or rectangles, students divide the whole by lines to unequal parts. Another problem is the identification of the whole by solving word problems requiring the use of fractions. The analogical error can also occur when students solve similar types of tasks with percentages. Fuchs pointed out in the paper (Fuchs, 2015) that this type of error often occurs in mathematics teaching at high school. The upper secondary school examination in mathematics in Czech Republic contain the task: 800 people came to the concert, which was a quarter of people more than the organizers expected. Determine how many people the organizers expected. Only 33.4 % of students solved this task correctly. In our research we gave to students in the first class at high school the analogical task adapted to the school environment in which number of people 800 was replaced with number of students 80 (see figure 3). The most common students' mistake is that one quarter does not count from the original whole but from the larger number and then subtract it from this number. Figure 3 shows the typical students' wrong solution.

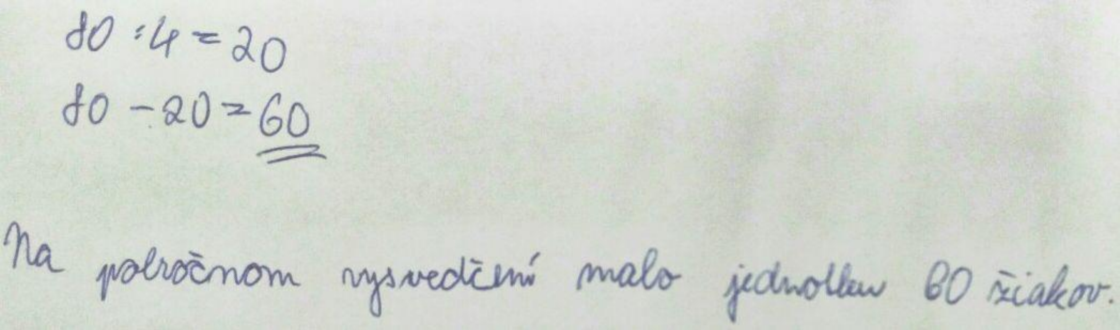


Figure 3 The most common students' wrong solution

When students investigate relationships between two variables they often analyse values written in tables. They can't characterize speed of changes of variable values and determine the type of dependence between variables. They prefer linear dependence without justification. Graphical representations are useful tool for better understanding and characterizing the type of dependence between variables. Diagrams in which the values of two variables are displayed next to each other can help students understand the graphs of functions built in the coordinate system. We have tried to take these aspects into account when creating an interactive application for the use of linear dependence to express the temperature in two scales (see figure 5).

4 EXAMPLES OF PROVIDING MINIMAL FEEDBACK IN MATHEMATICS LEARNING MATERIALS

The minimal feedback gives the student brief information about correctness or incorrectness of his/her answer. This simple feedback can also stimulate the student to think about each step of solution of the task or main idea which the student used for design a way how to solve the task. If the student get no feedback then he/she can make this mistake in the solving similar tasks. The absence of feedback becomes even more evident by doing homework. The minimal feedback may provide to the teacher in the learning process quick overview about students' success in their learning. Then the teacher can provide more effective help to students in need.

The minimal feedback may not only be based on a numerical value check, but it may also include a more complex evaluation of student's work results. The program Geogebra offers opportunities to control the visibility of objects that can also be used to provide feedback (Hohenwarter & Preiner, 2007). As an example, we selected the checking of correctness of a triangle construction which the student have to create in the program Geogebra. Assignment of the task: *Construct triangle ABC when given* $a = 5$ *cm*, $v_a = 6$ *cm*, $v_b = 4$ *cm.*

Construction of triangle *ABC* is displayed in figure 4. Figure 4 also contains auxiliary shapes which were used in the construction of triangle *ABC* and information for the student about correctness of the construction.

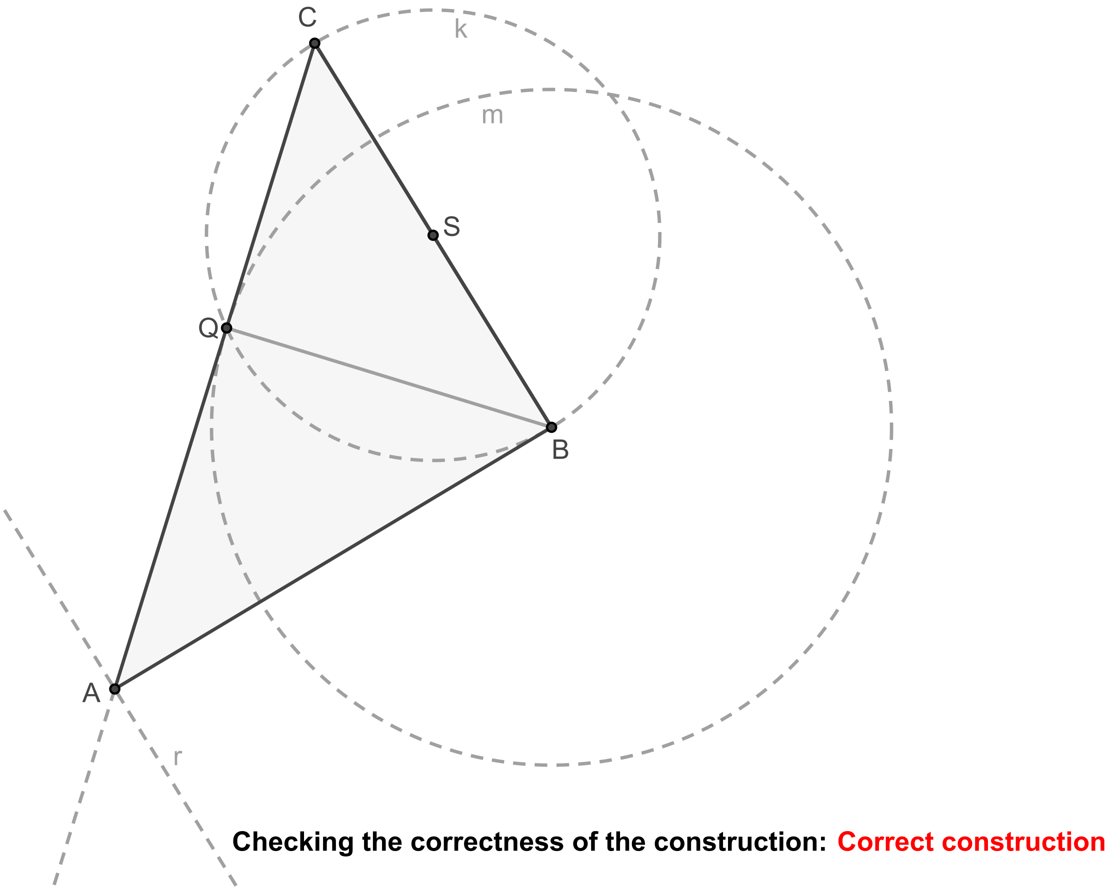


Figure 4 The construction of triangle *ABC* with minimal feedback

The segment *BQ* is the altitude to the side *b*. The construction of triangle *ABC* can start with constructing the altitude *BQ*. Another way of the construction of triangle *ABC* can be based on the construction of the triangle *QBC* using Thales' circle. The divergence of the solution process makes even more complicated the providing of feedback to errors in way of solving the task. Our design for providing the minimal feedback is only focused on checking of correctness of the resulting construction. If the corresponding elements in constructed triangle *ABC* are of the required sizes, then we can assume that the task is solved correctly. The student will not know in the case of negative outcome of feedback where he/she has made a mistake in the solution process, but he/she will know that it is necessary to edit the procedure for solving the problem or to ask for help. As a negative aspect of this design of feedback, we see the fact that a case might appear that the student would use the correct procedure to solve a task but he/she built some shape in the Geogebra environment incorrectly. Since triangle *ABC* does not have the required properties, so this solution will be considered incorrect. In the case of negative feedback, the teacher can provide a more accurate feedback to the students' solution process.

The second example is implemented in the program Geogebra. The interactive learning environment enables students to investigate linear dependence between quantities. Dynamic environment provides means for work with various representations: dynamic diagram, table and function rule. The diagram enables students to find a temperature expressed in °F corresponding to the temperature expressed in °C using sliders. Visualization using the built-in diagram (see figure 5) allows the student to easily determine the corresponding values of the variables and better understand the assignment of values of the dependent variable to values of the independent variable. Diagrams of this type are a suitable propaedeutic to understand function graphs constructed in coordinate system. Students are asked to enter whole numbers into the table. Specifying of the temperature in the last column requests to discover a linear dependence between temperatures expressed in two scales.

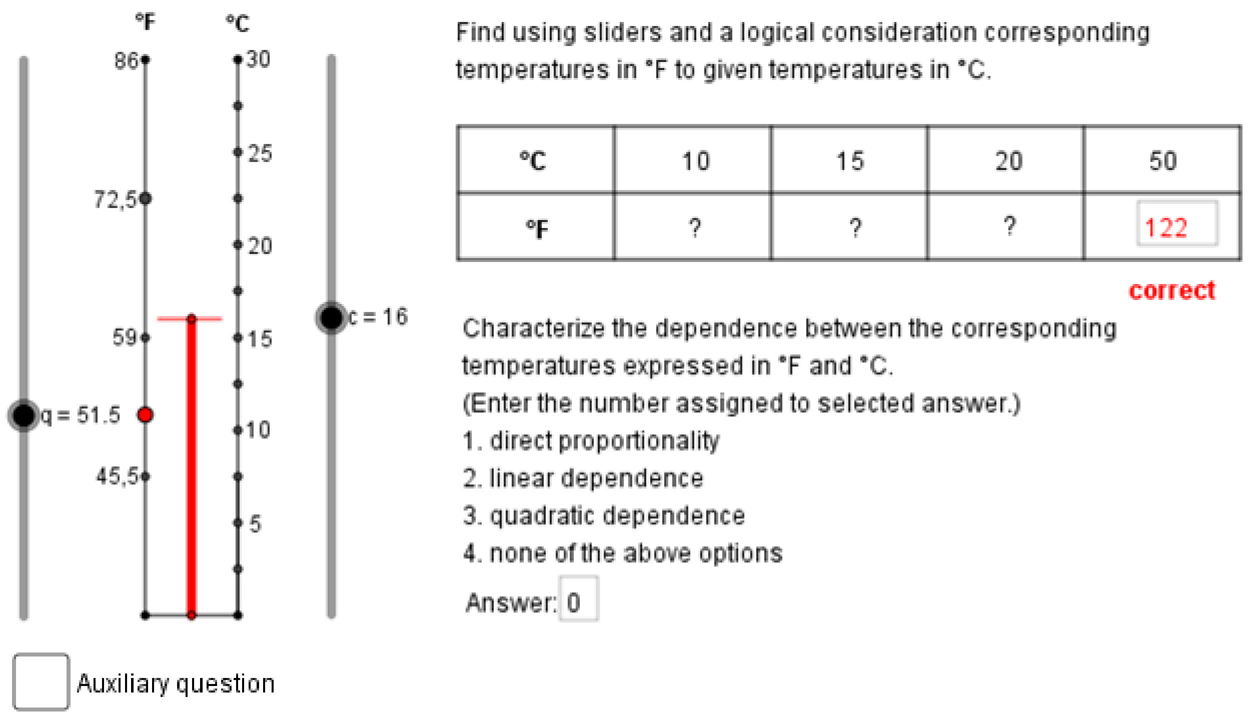


Figure 5 Mathematics learning environment for investigation of the functional dependence

The learning environment provides minimal feedback for written results. If a student is not able to identify a type of dependence between quantities, he can use the auxiliary question attached to a check box. The question directs student's attention on differences between the adjacent values in the table. Selection of the correct answer (2) causes displaying a new task which requires creation of the rule of the investigated linear function.

5 IMPLEMENTATION OF MORE EFFECTIVE FEEDBACK SUPPORTING FORMATIVE ASSESSMENT

When thinking about design and implementation of effective feedback supporting formative assessment it is suitable to focus on the critical parts of the learning content. These parts can be source of typical student's errors and misconceptions. In our research we focused on various topics, for example fractions, percentages, functions, probability. The example is selected from teaching word problems focused on fractions and percentages. Identification of a whole by calculation and comparison parts of whole is source of typical errors in this topic. This type of errors was highlighted in publication focused on evaluating results of upper secondary school examination in mathematics in Czech Republic (Fuchs, 2015).

The example is implemented in spreadsheet environment. Tasks which are the main part of feedback are placed on individual sheets. Figure 6 illustrates the first task and control button Evaluation to call reaction of the system. The student has to write result to cell with yellow background. Button New erases text field. Typical error is the calculation a quarter of 80 and subtraction this number from 80. If the student writes the incorrect result 60, he/she obtains the hint displayed in figure 6.

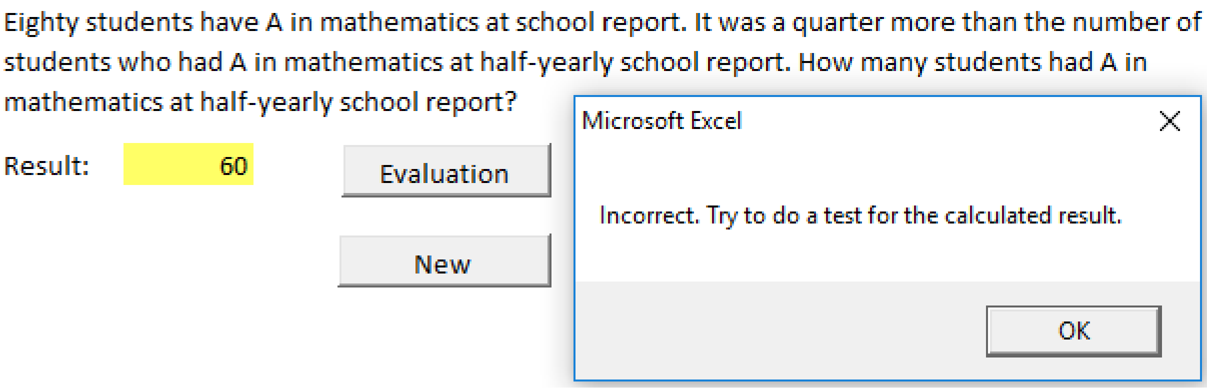


Figure 6 Part of worksheet with the first task

Writing the typical incorrect result will cause opening a new sheet containing the reformulated task: *If the number x is increased by a quarter of this number, we get the result 80. Specify the number x*. The assignment of the reformulated task is based on processual model and it suggests to the student how to complete the equation: $x + x/4 = 80$. The student should realize a relation between the solution of the reformulated task and the solution of the original task. If the student solves the reformulated task correctly, he/she will get an additional task analogous to the first task: 300 students attended the school carnival. It was a third less than expected. How many students did they expect at the school carnival? Correct solution of the first or the additional task leads to the providing a control task where part of the whole is expressed in percent: The price of the product after a 10% price increase was 242 €. What was the price of the product before the price increase? Feedback for incorrect results not related to typical error recommends the student to ask for help a teacher.

6 RESULTS AND DISCUSSION

Tasks from the described worksheet were used in the first class at high school (20 students). The average success of the students in the first task (see figure 6) was 15 % (3). The reformulated task solved correctly 50 % (10) of the students. The success of the students in the control task focused on percentages was the best (65 %, 13). The results do not point to unambiguous conclusion that the reformulated task helped students to understand incorrect solution of the first task. 20 % (4) of the students solved the reformulated task incorrectly but they solved the control task correctly. It can be assumed that the students already solved this type of tasks on percentages and they remember the algorithm. The correct students' solutions were based on the use of the formula $100 \cdot p/110$, where variable p represents price of the product after a 10% price increase. The analysis of the students' solutions shows that this knowledge has a formal character, because the students can not apply it in analogous situations when the percentages are replaced by fractions.

We designed feedback in the interactive learning materials intuitively based upon our experience as teachers and as problem solvers. Effectiveness of feedback depends on individual students' abilities, problem difficulty and students' experience with the solving selected type of tasks. We assume that providing simpler auxiliary tasks or reformulated tasks is suitable type of feedback stimulating conceptual understanding.

A better understanding of this type of word problem solving should be shown after assignment of the similar tasks after a longer period of time for the same students. In our next research, we are planning to investigate students' success in solving this type of word problems on a selected sample of high school students. The students will solve the tasks placed in an interactive worksheet with implemented feedback in the first stage of the experiment. In order to evaluate the solution process of tasks, the students will be asked to write the auxiliary calculations on the paper. The results of these students will be compared with the results of a test containing the same type of word problems that the same students will solve after a longer time period (at least 5 months). The first working hypothesis will include the statement that students will achieve greater success in solving tasks in the test than in solving tasks in the worksheet used in the first stage of the experiment. We will plan to use Mann–Whitney U test for statistical verification of the first hypothesis. For evaluation of results, the students who will do the typical error in solving the first task in the worksheet will be split into groups according to correctness of solution of the reformulated and additional task. The second working hypothesis will be aimed at the students who will do the typical error in solving the first task in the worksheet, but then they will solve the reformulated and additional task correctly. The second working hypothesis will include the statement that these students will achieve an average success at least 80% in the test containing the similar word problems. Another benefit of the experiment may be empirical findings obtained from a detailed qualitative analysis based on a comparison of students' solutions in the worksheet and in the test for different groups of students created according to their success in solving tasks in the worksheet.

CONCLUSION

Different types of feedback implemented in the mathematical digital learning materials were presented in the article. If the interactive learning activities are used directly in lessons in mathematics then teacher can provide feedback through appropriate arguments and auxiliary questions. Teachers have opportunities to analyse the variety of different students' ideas and strategies and use them to provide a bridge between their students' thinking and mathematical understanding. Teacher's learning experience allows him/her to provide students more advanced types of feedback focused on student's attention to his/her mistakes, and to direct his/her thought process to understanding of problem solving.

Providing effective feedback in an interactive learning environment is a complex problem. It requires a consistent analysis of students' errors and their reasons. Limited possibilities of the use of feedback come to the forefront especially in eliminating misconceptions. Several researches (Perrenet & Groen, 1993) demonstrate that initial student misconceptions are extremely difficult to correct using hints. Increasing feedback efficiency could ensure combining feedback with teacher help.

ACKNOWLEDGEMENTS

The article was created with support of the VEGA project no. 1/0265/17: Formative assessment in science, mathematics and informatics.

This article was created in the framework of the National project IT Academy – Education for the 21st Century, which is supported by the European Social Fund and the European Regional Development Fund in the framework of the Operational Programme Human Resources.

REFERENCES

Key data on learning and innovation through ICT at school in Europe. [online]. EACEA P9 EURYDICE, European Commission, 2011, [cit. 2017-08-11]. Available from: http://eacea.ec.europa.eu/education/eurydice/documents/key_data_series/129en.pdf.

Fuchs, E. (2015). *Současný stav maturity z matematiky.* [in Czech]. Retrieved 20 June 2017, from https://www.jcmf.cz/sites/default/files/Z-6_AF-LXIV_E-Fuchs_Soucasny_stav_maturity_z_matematiky.pdf

Goos, M., Galbraith, P., Renshaw, P. & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior, 22*(1), 73-89. ISSN 0732-3123.

Hähkiöniemi, M. (2013). Teacher's reflections on experimenting with technology-enriched inquiry-based mathematics teaching with a preplanned teaching unit. *Journal of Mathematical Behavior, 32*(3), 295-308. ISSN 0732-3123.

Hohenwarter, M. & Preiner, J. (2007). Dynamic Mathematics with GeoGebra. *The Journal of Online Mathematics and Its Applications*, *7*, article ID 1448. Retrieved 22 June 2017 from http://www.maa.org/external_archive/joma/Volume7/Hohenwarter.

Keeley, P. & Tobey, Ch., R. (2011). *Mathematics Formative Assessment*. Corwin a Sage Company, 234 p. ISBN 978-1-4129-6812-6.

Koreňová, L. (2015). *Digitálne technológie v školskej matematike*. [in Slovak]. Univerzita Komenského v Bratislave, Bratislava. 112 p. ISBN 978-80-8147-025-7.

Marshall, C., J. (2013). *Succeeding with inquiry in science and math classrooms*. NSTApress Arlington, USA, 162 p., ISBN 978-1-4166-1608-5.

McKendree, J. (1990). Effective feedback content for tutoring complex skills. *Human-Computer interaction*, *5*, 381−413.

Perrenet, J. & Groen, W. (1993). A hint is not always a help. *Educational Studies in Mathematics*, *25*(4), 307–329. ISSN 0013-1954.

Polya, G. (1957). *How to solve it*. Princeton University Press. Retrieved 5 April 2017 from https://notendur.hi.is/hei2/teaching/Polya_HowToSolveIt.pdf.

Prextová, T. (2015). Adaptive testing in practice. *ICTE Journal*, *4*(2), 37−49. ISSN 1805-3726.

Rocard, M. (2007). Science Education NOW: *A Renewed Pedagogy for the Future of Europe*. Retrieved 16 February 2017 from http://ec.europa.eu/research/science-society/document_library/pdf_06/report-rocard-on-science-education_en.pdf.

Sarabando, C., Cravino, J., P. & Soares, A., A. (2016). Improving student understanding of the concepts of weight and mass with a computer simulation. *Journal of Baltic Science Education, 15*(1), 109−126. ISSN 1648-3898.

Žilková, K. (2009). *Školská matematika v prostredí IKT*. [in Slovak]. Univerzita Komenského v Bratislave, 138 p. ISBN 978-80-223-2555-4.